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A trapezoidal intuitionistic fuzzy optimization approach for crashing a budget constrained project

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ABSTRACT

While the classical fuzzy applications have gained popularity in many different areas, intuitionistic fuzzy set theory emerged as an advancement to better identify various complex circumstances such as the hesitancy of a decision maker under uncertainty. In this study, we propose a project scheduling model in which the allocated budget for crashing is treated as an intuitionistic fuzzy number. With the intuitionistic fuzzy approach, the uncertainty inherent in the project completion time as well as the hesitancy of a decision maker in using the budget amount are integrated into decision making process. The proposed model numerically provides more information than that of the classical ones and better represents the complexity of decision making related to time and cost in project management. Decision makers are provided not only with an optimal schedule but also with a determined degree of satisfaction/dissatisfaction related to the budget and project completion time. Our experimental results show that the decision makers can substantially save money with some delays in the project completion time. Thus, a broader perspective encompassing the satisfaction/dissatisfaction level of a decision maker is presented to schedule a project under uncertain conditions.

1. Introduction

A project is defined as a "temporary endeavor undertaken to create a unique product or service" [37] (PMI, 2008). Projects are the cornerstones of the development in economic dimensions of the countries. Throughout history, there have been projects of great magnitude and consequences like the Egyptian Pyramids and the Great Wall of China. Ever since, project management has gained more and more momentum, according to the forecast of the Project Management Institute, project-oriented sectors will contribute more than $ 20 trillion to GDP by 2027, and employers will require nearly 88 million workers to work in project management-related professions [36].

Project management's main components can be considered as planning, scheduling, and controlling, which need to be handled with a systems approach for the successful completion of a project [25]. Scheduling as one of the main pillars of project management is the laying out of the project activities in the required time order, taking into consideration the expected completion times of activities and the resources [11]. Herroelen and Leus [18] reviewed the fundamental methods for project scheduling under uncertainty some of which are reactive and robust (proactive) scheduling, stochastic and fuzzy project scheduling, and uncertainty project scheduling with deterministic network evolution structure. Zhu et al. [46] worked on the disruption of an ongoing project. They focused on resource-constrained cases with the regular finish–start precedence constraints and proposed a classification scheme to find solutions in case of disruptions. Simulation-based algorithms like ant colony optimization (ACO) and Monte Carlo (MC) technique have been used for the stochastic project crashing problems by Bowman [8] and gradient estimators developed for PERT (Program Evaluation and Review Technique) networks were used by Aghaie and Mokhtari [1]. Li et al. [27] studied the project crashing problem which has ambiguity in task durations. They focused on minimization of the task delays as their objective and introduced a robust mixed integer programming model.

It is possible to see different types of objective functions in optimization models developed for project scheduling. Time-based objective functions aim to minimize the completion time of the project. As an example, Creemers [10] introduced a model that minimizes the completion of the project by using lateness, tardiness, and earliness as time-based objective sub-functions. Budget on the other hand is also a key element of the problem. Therefore, another objective function is cost-based, which deals with the minimization of the project cost. There

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are many kinds of research considering both time and cost simultaneously as a tradeoff problem. Vanhoucke & Debels [42] discuss some extensions and heuristic procedures for discrete time/cost tradeoff problems. In a more recent study, Hajiagha et al. [17] develop a time, cost, and risk tradeoff model for a COVID-19 supply chain project.

The project scheduling problem (PSP) is mainly about the optimization of time and cost. Thus, PSP deals not only with the project time but also with the resource allocation and cash flow throughout the project lifetime. PSP organizes the activities to find an optimal solution concerning the time and money invested in the management of the project[35]. Brčić et al. [9] introduce three specific solution approaches for project scheduling problems as predictive, proactive, and reactive strategies. The predictive approach solves PSP as a deterministic problem. The proactive strategy deals with the activity duration and produces a robust baseline. The reactive strategy reschedules the original schedule after an unexpected event happens.

Project scheduling tasks are sometimes highly unpredictable, which can cause delays. This uncertainty may arise as a result of deviations from estimations, a lack of resources, delayed material deliveries, changes in readiness times and due dates, new activities or modifications in the project scope, delays due to bad weather conditions, and so on. Due to missed due dates and deadlines, resource inactivity, and higher work-in-process inventory as a result of repeated rescheduling, a disrupted schedule incurs higher expenses [18]. One way to overcome the drawbacks of uncertainty is using fuzzy linear optimization. It is the extended version of classical optimization, which has fuzzy parameters and variables[4]. Bellman and Zadeh [7] proposed an experimental method for the optimization of decision-making with fuzzy sets. Zimmermann [47] introduced an optimization approach with linear programming using fuzzy objectives and constraints which basically set the membership functions as the solution of each objective/constraint. Fuzzy optimization models are more flexible and can provide better results for real-life engineering problems with uncertainties. Hence, the problems that contain fuzzy parameters have been extensively studied and gained popularity recently. We see fuzzy applications in many different areas such as facility location [43], transportation problems (Singh et al., 2021) [40], smart manufacturing [32], project scheduling [23,24] among many others.

In project scheduling, project crashing is one of the three compression methods of shortening project duration besides fast-tracking and substitution methods. The project crashing method is based on extra resource allocation to an activity duration that is intended to be reduced. This activity time–cost relationship mostly becomes linear, which means that cost increases or decreases linearly related to the time passed [6]. Regarding fuzzy project crashing Liu [30] proposed a project planning model in which fuzzy activity times are considered. Lin [28] merges fuzzy numbers with statistics to propose a solution for the critical path of an activity network with project crashing. Ehsani et al. [13] propose a fuzzy multi-objective linear programming method for project crashing which aims to develop a more realistic project planning.

While the fuzzy applications widened in many different areas including project scheduling, the theory has evolved to cover various circumstances that cannot be explained by classical fuzzy approach. One such advancement is intuitionistic fuzzy set (IFS) theory which was introduced by Atanassov [5]. In this theory, an element is given not only with a membership function, which refers to the degree of acceptance but also with a non-membership function, which refers to the degree of rejection, such that the sum of them can be less than 1. This new approach enables us to explain the hesitancy degree due to the lack of information in the decision-making process. Being the first to apply intuitionistic fuzzy theory in optimization, Angelov [2] introduces an intuitionistic fuzzy optimization (IFO) problem and proposes a scheme to convert the model into a crisp (non-fuzzy) one with some illustrative examples. Due to their flexibility in handling uncertainty, IFS gains popularity day by day in many areas including optimization [16,29,39] decision making [34,31,44], pattern classification [26,41,14] and risk evaluation (Huang et al., 2022), [20] [21,45]. Among some practical studies presenting different perspectives, Meng et al. [33] analyze intuitionistic linguistic preference relations and propose an approach to provide intuitionistic preference information to better describe preferred and nonpreferred degrees of the decision makers. Hu et al. [19] develop a multi-criteria decision making method using intuitionistic fuzzy sets to rank doctors. Kaur and Singh [22] present an application of an intuitionistic fuzzy optimization technique in a multiobjective vendor selection problem. Deeming inaccuracy and ambiguity in the data, Santos et al. (2021) [38] use intuitionistic fuzzy data to solve data envelopment analysis models more realistically. Ahmadini [3] uses the concept in logistic regression models to handle vagueness and hesitation simultaneously. To deal with the uncertainty of the parameters involved in a graph, Dinar et al. [12] identifies some of the distance and degree-based indices used in intuitionistic fuzzy graphs.

Even though intuitionistic fuzzy set theory has been applied in many different areas, we did not come across any such study in project scheduling. Considering the decision maker's inherent hesitancy, especially towards the amount of budget for project scheduling, we put forward a new approach in which intuitionistic fuzzy concept is for the first time integrated into budget-constrained project scheduling to better represent the real-life situations. In real life, when a project will be completed and how much money should be spent throughout the process is naturally ambiguous. Thus, in our study, we propose a project scheduling model in which the allocated budget is treated as an intuitionistic fuzzy number owing to the hesitancy of a decision maker in the amount of budget to be used. With the intuitionistic fuzzy approach, the uncertainty inherent in the project completion time as well as the hesitancy of a decision maker in using the budget amount are integrated into decision making process. The proposed model numerically provides more information than that of the classical ones since it better represents the complexity of decision making related to time and cost in project management. For a given solution, the model provides information not only about the degree of satisfaction but also about the degree of dissatisfaction of a decision maker, which is not explained by classical fuzzy approach. So, with the help of the model, one can determine the optimum project completion time together with the level of satisfaction and dissatisfaction of a decision maker. Thus, considering the probable hesitancy of a decision maker, the model offers more alternatives than those obtained by the classical approaches to make a better decision. The main contributions of this study can be summarized as follows:

- For the first time intuitionistic fuzzy concept is utilized in budget-constrained project scheduling.
- Uncertainty in the project completion time as well as the hesitancy of a decision maker in using the budget amount are integrated into decision making process.
- Optimal solution of a project scheduling problem is given with both satisfaction and dissatisfaction levels relating to budget usage and associated project completion time.
- A more realistic model with more alternatives has been made available to make a better decision under uncertain conditions.

The remaining sections of the paper are organized as follows. Section 2 presents the basic concepts of intuitionistic fuzzy set theory and interval arithmetic. Section 3 discusses the development of intuitionistic fuzzy models originating from a classical project crashing model. Section 4 provides an algorithm to solve the proposed model. Section 5 describes numerical examples and solution outcomes and finally Section 6 summarizes and concludes the study.

2. Preliminaries

2.1. Intuitionistic fuzzy sets

Definition 2.1. [5] *An intuitionistic fuzzy set (IFS) $\tilde{\tilde{A}}$ in a universe of discourse X is defined as an object of the form.*

$$\tilde{\tilde{A}} = \left\{ \left(x, \mu_{\tilde{\tilde{A}}}(x), \nu_{\tilde{\tilde{A}}}(x)\right) : x \in X \right\},$$

where the functions $\mu_{\tilde{\tilde{A}}} : X \rightarrow [0,1]$ and $\nu_{\tilde{\tilde{A}}} : X \rightarrow [0,1]$ are called membership and non-membership functions of $\tilde{\tilde{A}}$, respectively, and they define the degree of membership and the degree of non-membership of an element $x \in X$, respectively, and for every $x \in X$

$$0 \leq \mu_{\tilde{\tilde{A}}}(x) + \nu_{\tilde{\tilde{A}}}(x) \leq 1.$$

We denote the set of all intuitionistic fuzzy sets in X by $IF(X)$.

Definition 2.2. [5] *Let $\tilde{\tilde{A}} \in IF(\mathbb{R})$. The α-cut (level set) of $\tilde{\tilde{A}}$, denoted by $A(\alpha)$, is defined as follows:*

$$A(\alpha) = \left\{ x \in \mathbb{R} : \mu_{\tilde{\tilde{A}}}(x) \geq \alpha, \alpha \in (0,1] \right\}.$$

The 0-cut (level set) of $\tilde{\tilde{A}}$ is separately defined as,

$$A(0) = cl\left\{ x \in \mathbb{R} : \mu_{\tilde{\tilde{A}}}(x) > \alpha \right\},$$

where cl denotes the closure of the set.

Definition 2.3. [5] *Let $\tilde{\tilde{A}} \in IF(\mathbb{R})$. The β-cut (level set) of $\tilde{\tilde{A}}$, denoted by $A^*(\beta)$, is defined as follows:*

$$A^*(\beta) = \left\{ x \in \mathbb{R} : \nu_{\tilde{\tilde{A}}}(x) \leq \beta, \beta \in [0,1) \right\}.$$

The 1-cut (level set) of $\tilde{\tilde{A}}$ is separately defined as,

$$A^*(1) = cl\left\{ x \in \mathbb{R} : \nu_{\tilde{\tilde{A}}}(x) < 1 \right\},$$

where *cl* denotes the closure of the set.

Definition 2.4. [5] *Let $\tilde{\tilde{A}} \in IF(\mathbb{R})$. The (α,β)-cut (level set) of $\tilde{\tilde{A}}$, denoted by $A(\alpha,\beta)$, is defined as follows:*

$$A(\alpha,\beta) = \left\{ x \in \mathbb{R} : \mu_{\tilde{\tilde{A}}}(x) \geq \alpha, \nu_{\tilde{\tilde{A}}}(x) \leq \beta \right\}$$

where $0 \leq \alpha + \beta \leq 1$.

Definition 2.5. *If $\tilde{\tilde{A}} \in IF(\mathbb{R})$ satisfies the following conditions, then it is called as an intuitionistic fuzzy number in $\mathbb{R}$:*

1. $A(0)$ and $A^*(1)$ are bounded sets in $\mathbb{R}$.
2. $\tilde{\tilde{A}}$ is a normal intuitionistic fuzzy set, i.e., there exists at least an element $x \in \mathbb{R}$ with $\mu_{\tilde{\tilde{A}}}(x) = 1$ and $\nu_{\tilde{\tilde{A}}}(x) = 0$.
3. The function $\mu_{\tilde{\tilde{A}}} : X \rightarrow [0,1]$ is upper semi-continuous at any $x_0 \in \mathbb{R}$, namely, if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $f(x) < f(x_0) + \varepsilon$ whenever $|x - x_0| < \delta$.
4. The function $\nu_{\tilde{\tilde{A}}} : X \rightarrow [0,1]$ is lower semi-continuous at any $x_0 \in \mathbb{R}$, namely, if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $f(x_0) - \varepsilon < f(x)$ whenever $|x - x_0| < \delta$.
5. $\mu_{\tilde{\tilde{A}}}$ is a quasi-concave function, i.e., for every a, $b \in \mathbb{R}$ and $\lambda \in [0,1]$

$$\mu_{\tilde{\tilde{A}}}(\lambda a + (1-\lambda)b) \geq \min\left\{ \mu_{\tilde{\tilde{A}}}(a), \mu_{\tilde{\tilde{A}}}(b) \right\}.$$

6. $\nu_{\tilde{\tilde{A}}}$ is a quasi-convex function, i.e., for every a, $b \in \mathbb{R}$ and $\lambda \in [0,1]$

$$\nu_{\tilde{\tilde{A}}}(\lambda a + (1-\lambda)b) \leq \max\left\{ \nu_{\tilde{\tilde{A}}}(a), \nu_{\tilde{\tilde{A}}}(b) \right\}.$$

We will denote the set of all fuzzy numbers in $\mathbb{R}$ by $IF_N(\mathbb{R})$.

Definition 2.6. *An intuitionistic fuzzy number (IFN) of the form $\tilde{\tilde{A}} = (\overline{a}_1, a_1, a_2, a_3, a_4, \overline{a}_4)$ where $\overline{a}_1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq \overline{a}_4$ is said to be a trapezoidal IFN (TrIFN) if its membership and non-membership functions, respectively, are defined as follows* Fig. 1:

$$\mu_{\tilde{\tilde{A}}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}; & a_1 \leq x \leq a_2, \\ 1; & a_2 \leq x \leq a_3, \\ \dfrac{a_3 - x}{a_4 - a_3}; & a_3 \leq x \leq a_4, \\ 0; & otherwise, \end{cases}$$

and

$$\nu_{\tilde{\tilde{A}}}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - \overline{a}_1}; & \overline{a}_1 \leq x \leq a_2, \\ 0; & a_2 \leq x \leq a_3, \\ \dfrac{x - \overline{a}_4}{\overline{a}_4 - a_3}; & a_3 \leq x \leq \overline{a}_4, \\ 1; & otherwise. \end{cases}$$

2.2. $(\boldsymbol{\alpha},\boldsymbol{\beta})$-cuts of trapezoidal intuitionistic fuzzy numbers

The (α,β)- cut of a TrIFN $\tilde{\tilde{A}} = (\overline{a}_1, a_1, a_2, a_3, a_4, \overline{a}_4)$ is the set of all $x \in \mathbb{R}$ whose degree of membership is greater than or equal to α and the degree of non-membership is less than or equal to β. That is,

$$A(\alpha,\beta) = A(\alpha) \cap A^*(\beta)$$

where $A(\alpha) = [a_1 + \alpha(a_2 - a_1), a_4 + \alpha(a_3 - a_4)]$ and $A^*(\beta) = [a_2 + \beta(\overline{a}_1 - a_2), \overline{a}_4 + \beta(\overline{a}_4 - a_3)]$.

Since a TrIFN is normal $A(\alpha) \cap A^*(\beta)$ is a non-empty interval and by interval arithmetic, the (α,β)- cut of a TrIFN can be written as

$$A(\alpha,\beta) = [A_L(\alpha,\beta), A_R(\alpha,\beta)]$$

where $A_L(\alpha,\beta)$ and $A_R(\alpha,\beta)$ are left and right endpoints of the interval such that

$$A_L(\alpha,\beta) = max\{a_1 + \alpha(a_2 - a_1), a_2 + \beta(\overline{a}_1 - a_2)\},$$

$$A_R(\alpha,\beta) = min\{a_4 + \alpha(a_3 - a_4), \overline{a}_4 + \beta(\overline{a}_4 - a_3)\}.$$

The operations on intuitionistic fuzzy numbers can be done by using interval arithmetic.

Let $A(\alpha,\beta) = [A_L(\alpha,\beta), A_R(\alpha,\beta)]$ and $B(\alpha,\beta) = [B_L(\alpha,\beta), B_R(\alpha,\beta)]$ be the (α,β)- cuts of IFNs $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$, respectively. Then addition, subtraction, multiplication, division, and scalar multiplication of interval numbers

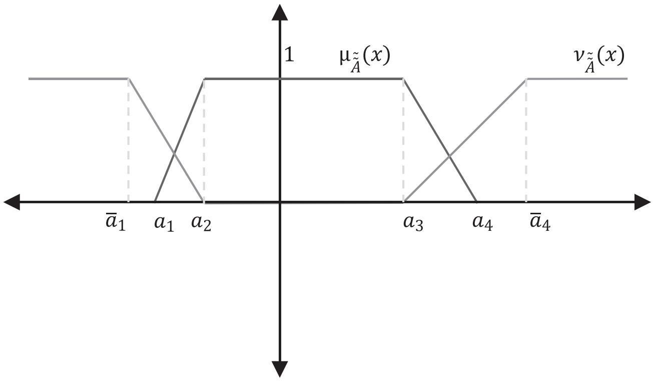

Fig. 1. A trapezoidal intuitionistic fuzzy number.

are described as follows:

Addition: $A(\alpha,\beta)+B(\alpha,\beta)=[A_L(\alpha,\beta)+B_L(\alpha,\beta),\ A_R(\alpha,\beta)+B_R(\alpha,\beta)]$.

Subtraction: $A(\alpha,\beta)-B(\alpha,\beta)=[A_L(\alpha,\beta)-B_R(\alpha,\beta),\ A_R(\alpha,\beta)-B_L(\alpha,\beta)]$.

Multiplication: $A(\alpha,\beta)B(\alpha,\beta)=[C_L(\alpha,\beta),C_R(\alpha,\beta)]$, where

$$C_L(\alpha,\beta)=\min\{A_L(\alpha,\beta)B_L(\alpha,\beta),A_L(\alpha,\beta)B_R(\alpha,\beta),A_R(\alpha,\beta)B_L(\alpha,\beta),A_R(\alpha,\beta)B_R(\alpha,\beta)\}$$

$$C_R(\alpha,\beta)=\max\{A_L(\alpha,\beta)B_L(\alpha,\beta),A_L(\alpha,\beta)B_R(\alpha,\beta),A_R(\alpha,\beta)B_L(\alpha,\beta),A_R(\alpha,\beta)B_R(\alpha,\beta)\}$$

Division: $A(\alpha,\beta)/B(\alpha,\beta)=[C_L(\alpha,\beta),C_R(\alpha,\beta)]$, where

$$C_L(\alpha,\beta)=\min\{A_L(\alpha,\beta)/B_L(\alpha,\beta),A_L(\alpha,\beta)/B_R(\alpha,\beta),A_R(\alpha,\beta)/B_L(\alpha,\beta),A_R(\alpha,\beta)/B_R(\alpha,\beta)\}$$

$$C_R(\alpha,\beta)=\max\{A_L(\alpha,\beta)/B_L(\alpha,\beta),A_L(\alpha,\beta)/B_R(\alpha,\beta),A_R(\alpha,\beta)/B_L(\alpha,\beta),A_R(\alpha,\beta)/B_R(\alpha,\beta)\}$$

provided that $0\notin B(\alpha,\beta)$.

Scalar Multiplication: For any real number k

$$kA(\alpha,\beta)=\begin{cases}[kA_L(\alpha,\beta),kA_R(\alpha,\beta)]; & k\geq 0,\\ [kA_R(\alpha,\beta),kA_L(\alpha,\beta)]; & k<0.\end{cases}$$

3. Model formulation

3.1. Budget constrained project crashing

Projects have their normal completion times with normal costs and activity durations. However, there are cases in which a project manager wants to complete the project earlier than its normal completion time by crashing activity times with some additional costs. A classic linear programming (LP) model to crash a project with a budget constraint can be given as follows.

Model 1: Budget Constrained Project Crashing Model

Indices and Sets

i,j	Activities 0,1,2,………..,(n + 1) (0 is dummy start node, n + 1 is dummy end node)
A(i, j)	Set of activity pairs having an immediate predecessor relationship {(0,1), (0,2), (1,4), (1,5), …, (n, n + 1)}

Parameters

t_i	normal duration (time) for activity i
Utt_i	upper bound for the crash time of activity i
CC_i	crash cost of activity i per unit time
B	available budget for project crashing

Decision Variables

s_i	start time of activity i
tt_i	amount of time by which activity i is crashed

Objection Function and Constraints

$$\textbf{\textit{Min}}\ Z=s_{n+1} \tag{3.1}$$

$$s_j-s_i\geq t_i-tt_i\quad\forall(i,j)\in \mathrm{A} \tag{3.2}$$

$tt_i\leq Utt_i\quad\forall i$ (3.3)

$$\sum\nolimits_i CC_i tt_i\leq B \tag{3.4}$$

$$s_i\geq 0, tt_i\ \geq 0\quad\forall \tag{3.5}$$

In the above formulation, the objective function (3.1) gives the minimum project completion time. Constraint (3.2) defines the time required between activities having an immediate predecessor relationship. Constraint (3.3) sets the activity time limits whereas Constraint (3.4) describes the available budget that can be used in crashing. Finally, Constraint (3.5) is used to define our variables.

3.2. Developing an intuitionistic fuzzy model for budget constrained project crashing

In real-life situations, due to incomplete information and uncertainty, the parameters used in Model 1 may not be that precise. This may occur due to unstable marketing situations, unpredictable setbacks in activity operations, or a change in project targets. The allocated budget for project crashing is inherently fuzzy since when exactly the project will finish and what additional costs will come up throughout the process is not definite. To deal quantitatively with such imprecise situations, we can consider the budget constraint as an IFN. Because of its versatility to handle more generic cases we use TrIFN in developing our fuzzy model.

First, we introduce additional parameters and decision variables to develop our intuitionistic fuzzy model as given follows.

Parameters

$\tilde{\tilde{B}}$	Trapezoidal intuitionistic fuzzy budget
B_R	The right endpoint value of (α,β)-cut of the intuitionistic fuzzy budget

Decision variables

α	Membership level (acceptance degree) for used budget
β	Non-membership level (nonacceptance degree) for used budget

Considering that the budget constraint in Model 1 is an IFN, we present the intuitionistic fuzzy version of the model as follows.

Model 2: Project Crashing Model with Intuitionistic Fuzzy Budget Constraint

$$\textbf{\textit{Min}}\ \tilde{\tilde{Z}}=s_{n+1} \tag{3.6}$$

$$s_j-s_i\geq t_i-tt_i\quad\forall(i,j)\in \tag{3.7}$$

$$tt_i\leq\ Utt_i\quad\forall \tag{3.8}$$

$$\sum_i CC_i tt_i \leq \tilde{\tilde{B}} \tag{3.9}$$

$$s_i \geq 0, tt_i \geq 0 \quad \forall \tag{3.10}$$

In model 2 budget is given as a fuzzy number as seen in Constraint (3.9) above. Here, we define the membership function and non-membership of

$\tilde{\tilde{B}} = (b_1 - d_1, b_1, b_2, b_3, b_4, b_4 + d_4)$ as follows:

$$\mu_{\tilde{\tilde{B}}}(x) = \begin{cases} \dfrac{x - b_1}{b_2 - b_1}; & b_1 \leq x \leq b_2, \\ 1; & b_2 \leq x \leq b_3, \\ \dfrac{b_4 - x}{b_4 - b_3}; & b_3 \leq x \leq b_4, \\ 0; & otherwise, \end{cases}$$

$$\nu_{\tilde{\tilde{B}}}(x) = \begin{cases} \dfrac{b_2 - x}{b_2 - b_1 + d_1}; & b_1 - d_1 \leq x \leq b_2, \\ 0; & b_2 \leq x \leq b_3, \\ \dfrac{x - b_3}{b_4 - b_3 + d_4}; & b_3 \leq x \leq b_4 + d_4, \\ 1; & otherwise, \end{cases}$$

where d_1 and d_4 are positive real numbers indicating the acceptable limit of the non-membership function of the budget. The (α,β)- cut of $\tilde{\tilde{B}}$ can be written as

$$B(\alpha,\beta) = [B_L(\alpha,\beta), B_R(\alpha,\beta)]$$

where

$$B_L(\alpha,\beta) = max\{b_1 + \alpha(b_2 - b_1), b_2 - \beta(b_2 - b_1 + d_1)\}$$

$$B_R(\alpha,\beta) = min\{b_4 + \alpha(b_3 - b_4), b_3 + \beta(b_4 - b_3 + d_4)\}$$

By using the (α,β)- cut of $\tilde{\tilde{B}}$ as given in (3.14) below, Model 2 can be converted to the following model, Model 2.a.

Model 2.a: Project Crashing Model using the (α, β)- cut of Intuitionistic Fuzzy Budget Constraint

$$\textbf{Min} \ \tilde{\tilde{Z}} = s_{n+1} \tag{3.11}$$

$$s_j - s_i \geq t_i - tt_i \quad \forall (i,j) \in A \tag{3.12}$$

$$tt_i \leq Utt_i \quad \forall \tag{3.13}$$

$$\sum_i CC_i tt_i \leq B_R(\alpha,\beta) \tag{3.14}$$

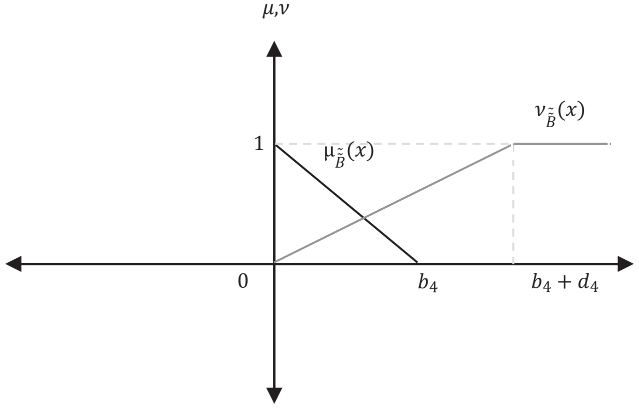

Fig. 2. Membership and non-membership function of intuitionistic fuzzy budget.

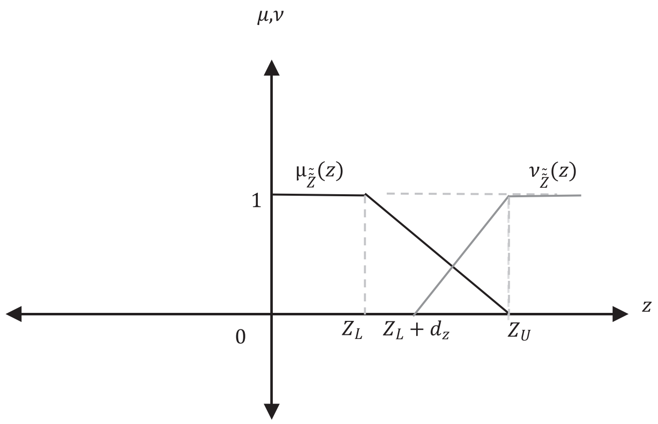

Fig. 3. Membership and non-membership function of the intuitionistic fuzzy objective function $\tilde{\tilde{Z}}$.

$$s_i \geq 0, tt_i \geq 0 \quad \forall \text{i} \tag{3.15}$$

where $B_R(\alpha,\beta) = min\{b_4 + \alpha(b_3 - b_4), b_3 + \beta(b_4 - b_3 + d_4)\}$.

Remark. The most suitable TrIFN budget constraint is of the form $\tilde{\tilde{B}} = (0, 0, 0, 0, b_4, b_4 + d_4)$ since a project manager normally would not be willing to allocate any budget for a project crashing but still could have to do so depending on developing conditions (See Fig. 2). In such a case, we obtain $B_R = \min\{(b_4 - \alpha b_4), \beta(b_4 + d_4)\}$.

Let Z_U be the optimal solution of Model 1 for the case where $B = 0$, and let Z_L be the optimal solution of Model 1 for the case where $B = b_4 + d_4$. Let us now define membership and non-membership functions for the objective function $\tilde{\tilde{Z}}$ as follows (see Fig. 3).

$$\mu_{\tilde{\tilde{Z}}}(z) = \begin{cases} 1; & 0 \leq z \leq Z_L, \\ \dfrac{Z_U - z}{Z_U - Z_L}; & Z_L \leq z \leq Z_U, \\ 0; & z \geq Z_U, \end{cases}$$

$$\nu_{\tilde{\tilde{Z}}}(z) = \begin{cases} 0; & z \leq Z_L + d_z, \\ \dfrac{z - Z_L - d_z}{Z_U - Z_L - d_z}; & Z_L + d_z \leq z \leq Z_U, \\ 1; & z \geq Z_U, \end{cases}$$

Considering these intuitionistic fuzzy numbers, we can now present our ultimate model, Model 2.b, to be used in our applicatory calculations.

Model 2.b: Ultimate Intuitionistic Fuzzy Budget Constrained Project Crashing Model.

Table 1
Details of a sample project.

Activity	Immediate Predecessor	Duration (in days)
A	–	6
B	A	8
C	A	9
D	B,C	7
E	B,D	6
F	D,G	11
G	D	8
H	C,G	6
I	E,F	12
J	F,G,I	9
K	G,H,J	6
L	I,J	7
M	K	9
N	J,L,M	8

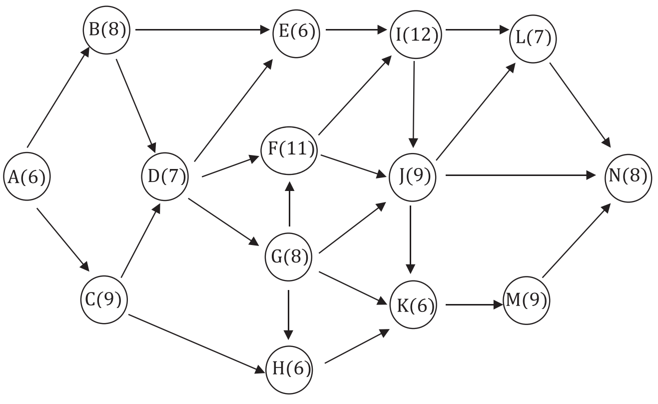

Fig. 4. AoN representation of the sample project.

$$\mathbf{Max}\ \ (\alpha - \beta) \tag{3.16}$$

$$s_{n+1} \leq Z_U - \alpha(Z_U - Z_L) \tag{3.17}$$

$$s_{n+1} \leq Z_L + d_Z + \beta(Z_U - Z_L - d_Z) \tag{3.18}$$

$$s_j - s_i \geq t_i - tt_i \quad \forall (ij) \in A \tag{3.19}$$

$$tt_i \leq Utt_i \quad \forall \tag{3.20}$$

$$\sum\nolimits_i CC_i tt_i \leq B_R(\alpha,\beta) \tag{3.21}$$

$$\alpha + \beta \leq 1 \tag{3.22}$$

$$\alpha \geq 0, \beta \geq 0 \tag{3.23}$$

$$s_i \geq 0, tt_i \geq 0 \quad \forall \tag{3.24}$$

where $B_R(\alpha,\beta) = \min\{(b_4 - \alpha b_4), \beta(b_4 + d_4)\}$.

Table 2
Crashing Data for Sample Project.

Activity	Normal		Crash		Allowable Crash Days	Crash Cost per Day ($)
	Time	Cost ($)	Time	Cost ($)		
A	6	5,000	4	7,000	2	1,000
B	8	6,000	5	7,500	3	500
C	9	7,500	5	9,500	4	500
D	7	4,000	6	5,500	1	1,500
E	6	3,000	4	4,500	2	750
F	11	9,000	7	13,000	4	1,000
G	8	7,000	6	8,500	2	750
H	6	3,500	4	6,500	2	1,500
I	12	8,000	9	11,000	3	1,000
J	9	6,000	7	10,000	2	2,000
K	6	4,500	5	6,000	1	1,500
L	7	5,500	5	7,000	2	750
M	9	7,500	6	9,000	3	500
N	8	8,500	5	11,500	3	1,000

Table 3
Start times, crashed times, and criticality of activities for Case 0.

Activity	Start Time	Crashed Time	Critical (+)/ Noncritical (-)
A	0	0	+
B	6.000	0	–
C	6.000	0	+
D	15.000	0	+
E	35.000	0	–
F	30.000	0	+
G	22.000	0	+
H	30.000	0	–
I	41.000	0	+
J	53.000	0	+
K	62.000	0	+
L	62.000	0	–
M	68.000	0	+
N	77.000	0	+
T	**85.000**	0	+

4. An algorithm for solution

We propose using the following algorithm to solve Model 2.

```
Start: ReadParameters
$p_b = intiutionistic\ fuzzy\ buget\ parameter(tolerance\ percentage\ in\ budget)$
$p_z = intiutionistic\ fuzzy\ objective\ function\ parameter(tolerance\ percentage\ in\ project\ completion\ time)$
$b_4 = \frac{available\ budget\ with\ tolerance}{1 + p_b}$
$d_4 = p_b b_4$
$d_z = p_z(Z_U - Z_L)$
    Step 1:FindZ_U
$B = 0$
$Solve\ Model1$
$Z_U = Optimal\ Z\ value\ of\ Model\ 1$
    Step 2:FindZ_L
$B = b_4 + d_4$
$Solve\ Model\ 1$
$Z_L = Optimal\ Z\ value\ of\ Model\ 1$
    Step 3:Define membersip and non-membership functions of budget
$\tilde{\tilde{B}} = (0,0,0,0,b_4,b_4 + d_4)$
    Step 4: SolveModel2.b
```

5. Application and results

5.1. Problem setting

Consider a project for which activities, immediate predecessor relationship between activities, and associated activity durations are given in Table 1.

Using activity on node representation (AoN), the corresponding network representation of the project is given in Fig. 4.

The data needed to crash the project is given in Table 2. In this respect, together with normal times and costs, where there is no

Table 4a

Project completion times, available and used budgets for Case 1.

	Model 1	**Model 2.b**
Available Max Budget ($)	25,500	TrIFN with$\widetilde{\widetilde{B}} = (0, 0, 0, 0, 23182, 25500)$
Used Budget ($)	25,500	10,825.516
Project completion time (days)	60	71.674
$\alpha(membersiplevel)$	0	0.533
$\beta(nonmembershiplevel)$	1	0.425

crashing, each activity has an associated crash time, crash cost, allowable crash time, and crash cost per unit time.

5.2. Solution results and discussion

We use the algorithm proposed in Part 4 and GAMS/CPLEX Version 12 [15] to find a solution. In this respect, we use $p_b = 0.1$ and $p_z = 0.1$ as our intuitionistic portions of budget and objective function values. In other words, we assume that the 10 percent of the budget and project completion time as well, are intuitionistic for the project manager. In our illustrative application, we use the following three cases with respect to budget availability.

Case 0. *There is no budget for crashing the project* ($Z = Z_U$).

The results for this base case, in which no crash budget is available and Z takes its upper value (Z_U), are obtained through our classical budget-constrained project crashing model, Model 1, and are presented in Table 3. We can see from Table 3, the start times and crashed times of each activity as well as whether the associated activity is critical or not. Of course, since there is no budget, the crashed times are 0 for all activities. Note that project completion time without any crash budget is 85 days, the highlighted start time of terminal activity T in Table 3. The designated critical activities are A, C, D, F, G, I, J, K, M, N, T.

Case 1. *There is enough budget for crashing the project such that earliest project completion time is achievable* ($Z = Z_L$).

In this case, we consider that there is enough money to crash the activities within the allowable limits such that Z takes its lower value (Z_L). Available budget, project completion times, and used budgets obtained through Model 1 and Model 2.b together with resulting $\alpha and \beta$ values are presented in Table 4a.

As seen in Table 4a, when there is a budget of $25,500, Model 1 yields 60 days of project completion time using all the 25,500 budget available. Naturally, budget acceptance degree α is 0, and budget nonacceptance degree β is 1 for Model 1 since all the available budget is used. On the other hand, Model 2.b gives approximately 72 days of project completion time using only 10825.516 (around 43 %) of the whole budget. The resulting budget acceptance degree α and nonacceptance degree β are 0.533 and 0.425 respectively. The results suggest that even though the project completion time in Model 2.b is around 12 days later, it might be preferable for the decision makers since around 14,674.484 (57 percent) of the available budget is saved.

Table 5a

Project completion times, available and used budgets for Case 2.

	Model 1	**Model 2.b**
Available Budget ($)	13,000	TrIFN with$\widetilde{\widetilde{B}} = (0, 0, 0, 0, 11818, 13000)$
Used Budget ($)	13,000	7222.147
Project completion time (days)	69.5	75.278
$\alpha(membersiplevel)$	0	0.389
$\beta(nonmembershiplevel)$	1	0.568

Activity start times and crashed amount of activity durations are presented in Table 4b, which also indicates the critical or noncritical status of each activity in the solution.

Case 2. *There is budget enough only for partially crashing the project such that project completion time is between* Z_U *and* Z_L ($Z_L < Z < Z_U$).

In this case, we assume that the available budget does not allow us to fully crash the activities within the allowable limits. However, depending on the budget we can partially crash the activities. Available budget, project completion times, and used budgets obtained through Model 1 and Model 2.b with resulting $\alpha and \beta$ values are presented in Table 5a.

As seen in Table 5a, when there is a budget of $13,000, Model 1 yields 69,5 days of project completion time whereas Model 2.b gives 75.278 days. Note that Model 1 uses all the available $13,000 of the budget while Model 2.b uses only $7222.147 (56 percent) of it. It is also noticeable that more of the available budget is used percentagewise in this case when compared to Case 1 with budget acceptance degree α and nonacceptance degree β being 0.389 and 0.568 respectively. Saving around 44 % of the available budget, Model 2.b can provide results more tempting for the decision makers even if the project completion time is 6 days longer than that of Model 1.

Activity start times and crashed amount of activity durations are given in Table 5b along with each activity's critical or noncritical status in the solution.

5.3. Sensitivity analysis

In this part of the study, considering that the budget is the most

Table 4b

Start times, crashed times, and criticality of activities for Case 1.

Activity	**Model 1**			**Model 2**		
	Start Time	Crashed Time	Critical (+)/ Noncritical (-)	Start Time	Crashed Time	Critical (+)/ Noncritical (-)
A	0	2.000	+	0	2.000	+
B	4	3.000	–	4.000	0	+
C	4	4.000	+	4.000	1.000	+
D	9	1.000	+	12.000	0	+
E	22	0	–	27.674	0	–
F	21	4.000	+	25.000	2.326	+
G	15	2.000	+	19.000	2.000	+
H	21	0	–	25.000	0	–
I	28	3.000	+	33.674	0	+
J	37	2.000	+	45.674	0	+
K	44	1.000	+	54.674	0	+
L	44	0	–	54.674	0	–
M	49	3.000	+	60.674	3.000	+
N	55	3.000	+	66.674	3.000	+
T	**60**	0	+	**71.674**	0	+

Table 5b
Start times, crashed times and criticality of activities for Case 2.

Activity	Model 1			Model 2		
	Start Time	Crashed Time	Critical (+)/ Noncritical (-)	Start Time	Crashed Time	Critical (+)/ Noncritical (-)
A	0	2.000	+	0	0	+
B	4.000	0	+	6.000	0	+
C	4.000	1.000	+	6.000	1.000	+
D	12.000	0	+	14.000	0	+
E	26.000	0	–	32.000	0	–
F	25.000	4.000	+	27.000	0	+
G	19.000	2.000	+	21.000	2.000	+
H	25.000	0	–	27.000	0	–
I	32.000	3.000	+	38.000	3.000	+
J	41.000	0	+	47.000	0	+
K	50.000	0	+	56.000	0	+
L	50.000	0	–	56.000	0	–
M	56.000	3.000	+	62.000	3.000	+
N	62.000	0.500	+	68.000	0.722	+
T	69.500	0	+	75.278	0	+

Table 6
Sensitivity of the project completion time to the budget.

Available Budget ($)	Model 1			Model 2.b		
	Minimum Project Completion Time (Days)	α	β	Minimum Project Completion Time (Days)	α	β
0	85	–	–	85	–	–
1,000	83	0	1	83.3	0.068	0.925
2,000	81	0	1	81.8	0.127	0.859
3,000	79.67	0	1	80.7	0.174	0.807
4,000	78.5	0	1	79.8	0.207	0.770
5,000	77.5	0	1	79.1	0.238	0.736
6,000	76.5	0	1	78.5	0.261	0.710
7,000	75.5	0	1	77.9	0.283	0.686
8,000	74.5	0	1	77.4	0.303	0.664
9,000	73.5	0	1	76.9	0.322	0.642
10,000	72.5	0	1	76.5	0.340	0.622
11,000	71.5	0	1	76.1	0.357	0.603
12,000	70.5	0	1	75.7	0.373	0.585
13,000	69.5	0	1	75.3	0.389	0.568
14,000	68.5	0	1	74.9	0.404	0.552
15,000	67.5	0	1	74.6	0.418	0.536
16,000	66.5	0	1	74.2	0.431	0.521
17,000	65.5	0	1	73.9	0.444	0.507
18,000	64.5	0	1	73.6	0.456	0.494
19,000	63.67	0	1	73.3	0.466	0.482
20,000	63	0	1	73.1	0.476	0.471
21,000	62.33	0	1	72.8	0.486	0.460
22,000	61.75	0	1	72.6	0.496	0.449
23,000	61.25	0	1	72.4	0.504	0.440
24,000	60.75	0	1	72.2	0.513	0.430
25,000	60.25	0	1	71.9	0.521	0.421
26,000	60	0	1	71.6	0.537	0.421
27,000	60	0	1	71.4	0.546	0.413

critical model input and one of our main constraints, we analyze how sensitive our objective function value (minimum project completion time) is to the given budget. Using incremental budget values, we get the results for Model 1 and Model 2.b as presented in Table 6.

As seen in Table 6, with increased values of budget we get shorter project completion times. However, it is noticeable that project completion time in Model 1 is more sensitive to budget than that of Model 2.b. That is, project completion time decreases more sharply in Model 1 than it does in Model 2.b. It makes sense since not all of the available budget is used in Model 2.b in line with the decision maker‘s designated satisfaction degree. It is also notable that α value gets larger and β value gets smaller with the increased amount of budget, which can be interpreted as the increase of acceptance level of budget usage for the decision maker when more budget is available for the project. We can also observe from the table that increasing the budget more than 26.000 does not have any effect on project completion time in Model 1 whereas it can still have some effect in Model 2.b, which can be attributed to the fuzziness in the latter.

Another parameter whose effect could be of interest to observe is the activity times (t_i) relating to constraint set defining the time required between activities having an immediate predecessor relationship. In fact, the effect of changes in activity times on the project completion time can be observed in our previous results since critical activities are already presented. It is known that any unit time increase in the durations of critical activities will cause a unit time delay in the project completion time, whereas increase in the durations of noncritical activities will have no effect on it. Thus, the decision makers should naturally be more concerned about the delays in the critical activities.

6. Conclusions

In this paper, we first develop a new intuitionistic fuzzy project crashing model by introducing the budget constraint as a trapezoidal intuitionistic fuzzy number. Then to make it computationally more applicable, we convert the intuitionistic fuzzy model into an equivalent version obtained by (α,β)-cuts and propose an algorithm for the solution. We apply the proposed approach to a budget-constrained project crashing model. In one of our case studies, in which there is enough budget for fully crashing the project such that the earliest project completion time is achievable, we observe that it is possible to save up to 57 % percent of the budget with a 12-day delay in project completion time when compared to the results of the classical model. In this case, the resulting degree of satisfaction (α) and the degree of dissatisfaction (β) are 0.533 and 0.425 respectively. In the other case, in which the available budget is more limited and allows only partial crashing, the model offers results with a budget savings of around 44 % with an extension of 6 days in project completion time. The resulting degree of satisfaction (α) and the degree of dissatisfaction (β) are 0.389 and 0.568 respectively. It is worth mentioning that in the first case the satisfaction degree is higher than the dissatisfaction degree contrary to the second case. This is because the percentage of the saved money is bigger in the first case. Such results may offer a broader perspective to decision-makers in terms of budget usage, project completion time, and utility level. That is, the decision-makers are not only provided with an optimal schedule and optimal budget usage but also with the degree of satisfaction and dissatisfaction related to the budget. Hence, the decision-makers can substantially save money with some delays in the completion time of the projects with uncertainties, which cannot be handled by the classical fuzzy approach.

As a future study, one can integrate the intuitionistic fuzzy approach also in activity times together with the budget to be used. Instead of intuitionistic fuzzy, a q-rung orthopair fuzzy set-based project crashing model might also be developed. Furthermore, q-rung fuzzy multiple criteria decision making methods can be integrated into our proposed approach to modeling and eliminating the drawbacks of human subjective decision-making which we usually come across in the real life.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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