Research Article

A mathematical model on liquefied natural gas supply chain with uncertain demand

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Received: 22 October 2019 / Accepted: 5 August 2020 / Published online: 11 August 2020

Abstract

This study focuses on the optimization of transportation and the storage of a Liquefied Natural Gas (LNG) supply chain. Liquefaction of the natural gas enables the suppliers to carry the gas by different modes of transportation to the customers by changing its state from gaseous state to liquid while decreasing its volume by 600 times. We consider the liquefaction, transportation, and re-gasification costs for the LNG supply chain and, propose a new model for the LNG supply chain, which minimizes the costs that may incur in the LNG supply chain. In the proposed model, sea, road, and pipeline transportation modes are taken into consideration. Unlike other models, in addition to the on-shore/stationary storage facilities at the storage sites, off-shore/ rented vessels (or floating holding tanks), are used to mitigate the demand uncertainty regarding the unexpected increase and decrease in customer demand. The LNG Supply Chain Model is first tested by 57 alternative test problems with randomly generated hypothetic data for each case and solved. Then the developed model is extended with normally distributed demand data which is generated through a Cycle Service Level (CSL) of 0.90 and solved by using GAMS CPLEX 24.1.3.

Keywords Liquefied natural gas · Supply chain management · Optimization · Linear programming

Mathematics Subject Classification 90B06

JEL Classification C61

1 Introduction

A supply chain includes all the components involved to satisfy a customer request. The supply chain includes manufacturers, suppliers, transporters, warehouses, retailers, and customers. Proper company decisions in the supply chain as a component can increase the supply surplus and profit. LNG is formed by reducing the Natural Gas (NG) to – 162 °C. The volume of natural gas shrinks about 600 times as the liquid passes through the gas phase. Hence, a high volume of NG can be stored in the liquid state by reducing the volume 600 times under high pressures. LNG makes it convenient to transport by ships and via cryogenic trailers in liquid form to be regasified at the final destination to the storage areas where it is not possible to transport the NG in pipelines technically and economically. Generally, LNG is preferred to NG because NG is not purified. LNG is purified from the oxygen, carbon dioxide, sulfur components, and water during the liquefaction process. This results in a purer and more efficient fuel than natural gas.

Supply chain decisions include the location and the capacity of the facilities, quantity of products to be stored in various locations, transportation/distribution

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SN Applied Sciences (2020) 2:1496 | https://doi.org/10.1007/s42452-020-03297-7

of these products, and finally how the information flows between these components. Correct decisions on LNG supply chain design will increase productivity and profitability. Good design of the LNG supply chain decreases the storage location, distribution, and network costs and increases the capabilities of the supply chain by improving the information flow along with the components with lower costs. The elements of the LNG supply chain are exploration and production of NG, liquefaction of NG, transportation of LNG, re-gasification of LNG, storage, and distribution. In this study, we present a new LNG supply chain model that tries to minimize the costs through these phases.

Ozelkan et al. [1] studied a framework for analyzing the key design parameters for LNG terminals. They developed a mixed-integer programming model with an application that includes the terminals of the LNG supply chain and transportation of LNG through the supply chain.

Gronhaug et al. [2] expressed the LNG inventory and routing problem in their study with inventory management and port constraints. They introduced a method to solve sub-problems for ship routes. In the study, a solution methodology was proposed to solve the problem much faster than commercial optimization software and to solve larger samples than the previous examples.

Fodstad et al. [3] presented a model for tactical planning and maximizing profits. In comparison with other models in the literature, a richer model is offered. Examples of how the added features change the solutions are presented. Rakke et al. [4] introduced a model with a case study for a company, which was one of the world's largest producers and distributors of LNG. They also defined a heuristic algorithm for the annual delivery schedule that solved the samples with reasonable time and good solution quality.

Khalilpour and Karimi [5] expressed the contract selection from the perspective of an LNG buyer company. They developed a mixed-integer linear programming model that helped the buyer to select the best combination of suppliers and contracts.

Stalhane et al. [6] introduced a model for the problem of ship routing with inventory management for LNG producer and distributor. Their objectives were to create a one-year delivery schedule by fulfilling their contractual obligations.

Goel et al. [7] presented a model to optimize both the production and re-gasification terminal programs together with inventory management. In their study, they expressed a mixed-integer programming model and heuristic algorithms to solve the problem.

Halvorsen-Weare and Fagerholt [8] were interested in the routing and scheduling problem of ships for carrying LNG. The problem was solved by dividing the problem into sub-problems. Different from the literature, time window constraints were added to the model.

Goel et al. [9] proposed a constraint-programming model for the LNG inventory and scheduling problem. Besides, they developed a heuristic algorithm to solve the problem. Compared to other studies in the literature, the introduced algorithm was faster than others. Halvorsen-Weare et al. [10] studied on the LNG vessel routing and schedule under uncertainty in sailing times and daily LNG production rates in their study.

Bagočius et al. [11] determined the most suitable terminal location selection in their study with 3 different multi-criteria decision-making methods.

Jokinen et al. [12] presented a mixed-integer programming model for small-scale LNG supply chain optimization. The model illustrated by a case study considered an LNG supply chain in Finland.

Shao et al. [13] developed a model based on the study of Goel et al. [9] in their work. They compared their study with the study of Goel et al. [9]'s.

Ghiami et al. [14] studied the design of the LNG distribution network in the Netherlands. The authors implemented an application on the inventory routing problem. Besides, they modeled this problem with both arc flow and path flow method and compared the results. They introduced the effects of deterioration on the total cost function.

Andersson et al. [15] introduced a model based on the study of Gronhaug et al. [2]. They proposed a new decomposition algorithm. Starting time of operation, loaded and unloaded quantity were taken into account for each task. Bittante et al. [16] developed a model as a general framework for fleet composition and ship routing within a given time horizon. Additionally, they presented a mixed-integer programming model for the optimization of a small-scale LNG supply chain with a case study in the Caribbean.

Misra et al. [17] took into consideration a supply chain optimization problem for packaged liquefied gaseous products, which considered a warehouse stock management. Raharjo and Sudibandriyo [18] proposed a model to minimize the total cost of the LNG supply chain, which took into consideration plant capacity with a case study.

Yazdi et al. [19] studied on an LNG ship routing and scheduling problem. The study aimed to minimize the total cost. They proposed a metaheuristic solution method to solve the problem. Ghiami et al. [20] presented a heuristic algorithm to maximize the total profit for an LNG supply chain. Sangaiah et al. [21] addressed a robust mixed-integer linear programming model for LNG sales planning over a given time horizon aiming to minimize the costs of the vendor. The studies in the literature are summarized in Table 1 below.

Considering the modeling details, as in Table 1, most of the studies about the LNG supply chain are in the field

Table 1 Summary of the studies in the literature about the LNG supply chain

Authors	Decisions	Modes of transport	Uncertainty	Case study	Members of chain	Model type
Özelkan et al. [1]	LNG supply chain	Sea transport	✓	✓	LNG plant, storage area, backlog, customer	MIP
Grønhaug et al. [2]	Inventory, routing	Sea transport	×	×	LNG plant, storage area, regasification plant	Path flow model, branch and price method
Fodstad et al. [3]	Inventory, routing, scheduling	Sea transport, Road transport	×	✓	LNG plant, storages, regasification plant, natural gas hub, customer	MIP
Rakke et al. [4]	Inventory, routing, scheduling	Sea transport, pipeline transport	×	✓	Liquefaction plant, storage area, regasification plant, customer	MIP, Rolling horizon heuristic
Khalilpour and Karimi [5]	Selection of LNG contracts	Sea transport	×	✓	LNG plant, storage areas, Re-gasification plant, customers	MILP
Stalhane et al. [6]	Inventory, routing	Sea transport, pipeline transport	×	✓	Liquefaction plant, storage area, regasification plant, customer	Multi start local search
Goel et al. [7]	Inventory, routing	Sea transport	×	✓	Liquefaction plant, storage areas, re-gasification plant, customers	MIP, Local neighbor search
Halvorsen-Weare and Fagerholt [8]	Routing, scheduling	Sea transport	×	✓	LNG plant, warehouse, regasification plant	MILP and decomposition method
Goel et al. [9]	Inventory, scheduling	Sea transport	×	✓	LNG plant, storage area, regasification plant	Constraint programming, iterative search heuristic
Halvorsen-Weare et al. [10]	Routing, scheduling	Sea transport	(Sailing time and daily LNG production rates)	✓	LNG plants, storage area, customers	Robustness strategies based on mathematical model formülation
Bagočius et al. [11]	Location	Sea transport	×	✓	LNG plant, warehouse, regasification plant	SAW, TOPSIS, COPRAS
Jokinen et al. [12]	LNG supply chain	Sea transport	×	✓	LNG plant, storage area, receiving port	MIP
Shao et al. [13]	Inventory, routing	Sea transport	×	✓	LNG plant, storage area, regasification plant	MIP, GRASP, MIP-based neighborhood search techniques
Ghiami et al. [14]	Inventory, routing	Sea transport	×	✓	Liquefaction plant, storage area, re-gasification plant, customer	Arc-flow model and path-flow model
Andersson et al. [15]	Inventory, routing	Sea transport	×	✓	LNG plant, warehouse, regasification plant	MIP, Decomposition algorithm, Branch and Bound method
Bittante et al. [16]	LNG supply chain	Sea transport	×	✓	LNG plant, receiving port	MIP
Misra et al. [17]	Inventory, routing	Sea and road transport	×	✓	Plant, warehouse, customer	MILP

Table 1 (continued)

Authors	Decisions	Modes of transport	Uncertainty	Case study	Members of chain	Model type
Raharjo and Sudibandriyo [18]	LNG supply chain	Sea and road transport	×	✓	LNG plant, regasification plant, and customer	LIP
Yazdi et al. [19]	Routing, scheduling	Sea and road transport	Ship failure during shipping	✓	LNG plant, regasification plant, and customer	MIP, Binary particle swarm optimization
Ghiami et al. [20]	Inventory,routing	Sea and road transport	×	✓	LNG plant, storage areas, and customer	Adaptive large neighborhood search algorithm
Sangaiah et al. [21]	LNG supply chain	Sea transport	×	✓	LNG plant	MIP, anovel metaheuristic algorithm
Dziri et al. [22]	LNG supply chain	Road transport	Demand uncertainty	✓	Supplier, storage areas, customer	Dynamic programming
Proposed Model	LNG supply chain, inventory	Sea, road and pipeline transport	Demand uncertainty	✓	LNG plant, regasification plant, storage area, rented vessel, hub and customer (two types)	LP

of inventory, routing, and scheduling. For example, Khalilpour, R., Karimi, I. A [5]. deals with the selection of LNG contracts and Bagočius et al. [11] develop a model with only the location of the facilities. Studies [2, 6, 7, 13–15, 17, 20] consider inventory and routing in their models while studies [8–10] and [19] includes routing and scheduling. Fodstad et al. [3] and Rakke et al. [4] consider inventory, routing, and scheduling. Studies [1, 12, 16, 18, 21] propose models on LNG supply chain.

Most of the studies assume that demand, inventory, service time, and other factors are fixed. They affect the supply chain design and modeling regarding the costs. While Halvorsen-Weare et al. [10], Bittante et al. [16], Yazdi et al. [19], Sangaiah et al. [21]'s studies include uncertainty, the others consider deterministic factors in their studies. Different from the other studies, Bittante et al. [16] consider both known and unknown factors in their models. In the probabilistic studies, generally, the data in concern are assumed to conform to normal distribution while developing models which are considering uncertainty.

We develop a model that considers all phases of the LNG supply chain and inventory management through different alternatives. The model includes liquefaction plants, on-shore/stationary storage facilities, off-shore rented vessels, regasification plants, hubs, and end-users unlike the other studies in the literature which include just some of the stages mentioned. As stated in Table 1, generally, the objectives of the models in the literature are the cost minimization. Most of the models include one type of transportation mode with a single phase of the LNG supply chain. For example, the studies [1, 2, 5, 7–16] and [21] include only one type of transportation modes. Studies [3, 4, 6, 17–19] and [20] include two types of transportation modes.

The LNG supply chain model that we propose includes three types of transportation modes while none of the other studies in the literature includes three or more.

The contributions of the model that we propose are the followings:

- The proposed LNG supply chain includes the off-shore/rented vessels (or floating holding tanks) for inventories in addition to the stationary storage areas to enhance the capability of the LNG companies to make use of extra tentative LNG storage to exploit market opportunities. Using rental vessels provides flexibility to the decision-makers in case there exists an unexpected demand change in the LNG market.
- The proposed model is multimodal with the different modes of transportation (sea, road, and pipeline) which are widely used by the global LNG industry.
- The model includes all phases of the LNG supply chain: liquefaction plants, on-shore/stationary storage facili-

ties, off-shore/ rented vessels (or floating holding tanks), regasification plants, hubs, and customers unlike the other studies including some stages of them.
- We develop a model considers a normally distributed LNG and NG demand data regarding the Cycle Service Level to cope with the adverse effects of uncertain demand in an NG/LNG supply chain.

2 The liquefied natural gas model

In this study, we propose a model includes a large-scale LNG receiving terminal, storage tanks, regasification units, and distribution infrastructure which will receive and store the imported/produced LNG in on-shore/stationary storage or off-shore/ rented vessels (or floating holding tanks), and distribute to the end-users either through a pipeline after being regasified or via cryogenic trailers in liquid form to be regasified at the final destination.

We develop a model related to the LNG supply chain defined before. The supplied NG is liquefied in liquefaction plants. After the liquefaction process, LNG is sent to the re-gasification facilities and on-shore stationary storage facilities according to the demands of the customers and the capacity of these plants. If the capacity of the storage area is exceeded, the rest of the LNG is sent to off-shore/ rented vessels (or floating holding tanks). LNG, which is sent to the re-gasification plant according to customer demands, is converted into gas here and sent to hubs via the pipeline transport. Liquid gas in off-shore/ rented vessels (or floating holding tanks) and on-shore stationary storage facilities is transported directly to customers by cryogenic trailers. In our model, it is assumed that the production capacity of the facilities and the capacity of the cryogenic trailers are sufficient. The steps are given in Fig. 1. The proposed mathematical model is formulated as a Linear Programming (LP) problem.

2.1 Problem description

Supply chain management is difficult and complex. In this study, the process of the LNG supply chain from liquefaction plants to distribution to customers is included. Satisfying customer demands with minimum cost is considered because of high supply chain costs. Additionally, different modes of transportation are included in the model while it is transported to the end-users. By considering the capacity of the on-shore stationary storage facilities may be insufficient, during high demand seasons, this model includes the off-shore/ rented vessels (or floating holding tanks) to increase the storage capacity. In this way, customer satisfaction and profits can be increased. The assumptions that we consider for the proposed model are as follows:

- In the proposed model, after NG transported to the liquefaction facility, it is converted to the liquid phase. Then the LNG transported to rented vessels, storage areas, and regasification plants. LNG is converted to the gaseous phase again in regasification plants. Then, the NG transported to the hub. In this stage, it is assumed that there is no gas loss in the system.
- It is assumed that the amount of gas/liquid entering the node is equal to the amount of gas/liquid leaving the node.
- In the model we assume that all the customer demand is satisfied. Thus, it is not allowed to have any shortage (backordering and lost sales are not allowed in the model).
- It is assumed that liquid gas collected in the storage area and rented vessels are sent to customers in the same period.

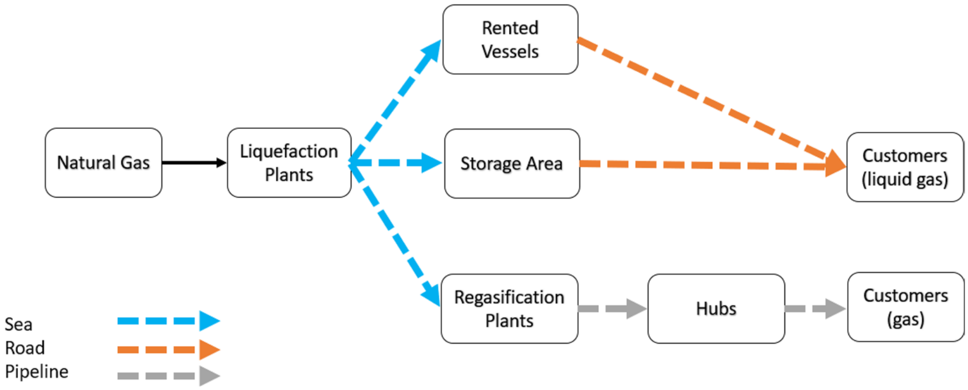


Fig. 1 LNG Supply chain in the proposed model

2.2 Sets

The sets which are used in the model are:

L: Liquefaction plants $L = \{1, 2, \ldots, l)$,

R: Re-gasification plants $R = \{1, 2, \ldots, r\}$,

T: Periods $T = \{1, 2, \ldots, t\}$,

K: off-shore/ rented vessels (or floating holding tanks) $K = \{1, 2, \ldots, k\}$,

V: Vessels $V = \{1, 2, \ldots, v\}$,

B: on-shore stationary LNG storage facilities $B = \{1, 2, \ldots, b\}$,

J: Hubs $J = \{1, 2, \ldots, j\}$,

M: LNG end-user (customer) $M = \{1, 2, \ldots, m\}$,

G: NG end-user (customer) $G = \{1, 2, \ldots, g)$.

2.3 Parameters

The parameters which are used in the model are:

$U_{l,t}$: Liquefaction cost of gas per unit at liquefaction plant l in period t.

$W_{k,t}$: Cost of storage of liquid gas per unit on off-shore/ rented vessel (or floating holding tank) k during the period t.

$C_{v,l,t}$: Cost of transport from liquefaction plant l to other plant gas per unit through vessel v in the period t.

$a_{l,b}$: Distance between liquefaction plant l and on-shore stationary LNG storage facility b.

$a_{l,r}$: Distance between liquefaction plant l and re-gasification plant r.

$a_{l,k}$: Distance between liquefaction plant l and off-shore/ rented vessel (or floating holding tank) k.

$a_{r,j}$: Distance between re-gasification plant r and hub j.

$a_{j,g}$: Distance between hub j and NG end-user (customer) g.

$a_{k,m}$: Distance between off-shore/ rented vessel (or floating holding tank) k and LNG end-user (customer) m.

$a_{b,m}$: Distance between liquid gas storage area b and LNG end-user (customer) m.

$h_{b,t}$: Holding cost of liquefied natural gas in liquid gas storage area b in the period t.

$e_{r,t}$: Cost of gasification at the re-gasification plant r in the period t.

s_t: Cost of road transport per unit in the period t.

n_t: Cost of pipeline transport per unit in the period t.

$DD_{m,t}$: The total demand for LNG customer m in the period t.

$DD_{g,t}$: The total demand for NG customer g in the period t.

$DS_{m,t}$: The total expected demands for LNG customer m in the period t.

$DS_{g,t}$: The expected demand for NG customer g in period t.

$\alpha_{b,t}$: Capacity of liquid gas storage area b in the period t.

$\beta_{k,t}$: Capacity of off-shore/ rented vessel (or floating holding tank) k in the period t.

2.4 Decision variables

The decision variables, which are used in the model, are:

$P_{l,t}$: Production amount at the liquefaction plant l in the period t.

$O_{l,k,t}$: Amount of liquid gas transported from the liquefaction plant l to the off-shore/ rented vessel (or floating holding tank) k in the period t.

$BA_{l,b,t}$: Amount of liquid gas transported from the liquefaction plant l to the storage area b in the period t.

$Q_{l,r,t}$: Amount of liquid gas transported from the liquefaction plant l to the re-gasification plant r in the period t.

$Y_{r,j,t}$: Amount of gas transported from the re-gasification plant r to the hub j in the period t.

$Z_{j,g,t}$: Amount of gas transported from the hub j to the NG end-user (customer) g in the period t.

$X_{k,m,t}$: Amount of liquid gas transported from the off-shore/ rented vessel (or floating holding tank) k to the LNG end-user (customer) m in the period t.

$F_{b,m,t}$: Amount of liquid gas transported from the storage area b to the LNG end-user (customer) m in the period t.

2.5 The model with deterministic demand

Subject to:

$$\begin{aligned}\text{Min } Z = {} & \sum_{l,t} U_{l,t}P_{l,t} + \sum\nolimits_{k,l,t} W_{k,t}O_{l,k,t} + \sum\nolimits_{l,b,t} h_{b,t}BA_{l,b,t} + \sum\nolimits_{v,l,t,b} C_{v,l,t}BA_{l,b,t}a_{l,b} + \sum\nolimits_{v,l,t,k} C_{v,l,t}O_{l,k,t}a_{l,k} + \sum\nolimits_{v,l,t,r} C_{v,l,t}Q_{l,r,t}a_{l,r} \\ & + \sum\nolimits_{r,l,t} e_{r,t}Q_{l,t} + \sum\nolimits_{k,m,t} S_tX_{k,m,t}a_{k,m} + \sum\nolimits_{b,m,t} S_tF_{b,m,t}a_{b,m} + \sum\nolimits_{r,j,t} n_tY_{r,j,t}a_{r,j} + \sum\nolimits_{g,j,t} n_tZ_{j,g,t}a_{j,g}\end{aligned} \tag{2.1}$$

$$\sum\nolimits_l Q_{l,r,t} = \sum\nolimits_j Y_{r,j,t}/600 \quad r = 1, .., R \quad t = 1, \ldots, T \tag{2.2}$$

$$P_{l,t} \geq \sum\nolimits_k O_{l,k,t} + \sum\nolimits_b BA_{l,b,t} + \sum\nolimits_r Q_{l,r,t} \quad l = 1, \ldots, L \quad t = 1, \ldots, T \tag{2.3}$$

$$\sum\nolimits_m X_{k,m,t} \leq \sum\nolimits_l O_{l,k,t} \quad k = 1, \ldots, K \quad t = 1, \ldots, T \tag{2.4}$$

$$\sum_{m} F_{b,m,t} \le \sum_{l} BA_{l,b,t} \quad b = 1, \ldots, B \quad t = 1, \ldots, T \tag{2.5}$$

$$\sum_{r} Y_{r,j,t} \ge \sum_{g} Z_{j,g,t} \quad j = 1, \ldots, J \quad t = 1, \ldots, T \tag{2.6}$$

$$\sum_{l} BA_{l,b,t} \le \alpha_{b,t} \quad b = 1, \ldots, B \quad t = 1, \ldots, T \tag{2.7}$$

$$\sum_{k} X_{k,m,t} + \sum_{b} F_{b,m,t} \ge DD_{m,t} \quad m = 1, \ldots, M \quad t = 1, \ldots, T \tag{2.8}$$

$$\sum_{l} O_{l,k,t} \le \beta_{k,t} \quad k = 1, \ldots, K \quad t = 1, \ldots, T \tag{2.9}$$

$$\sum_{j} Z_{j,g,t} \ge DD_{g,t} g = 1, \ldots, G \quad t = 1, \ldots, T \tag{2.10}$$

$$\sum_{l} P_{l,,t} \ge \sum_{m} DD_{m,t} + \sum_{g} DD_{g,t}/600 \quad t = 1, \ldots, T \tag{2.11}$$

$$P_{l,t}, O_{l,k,t}, BA_{l,b,t}, Q_{l,r,t}, X_{k,m,t}, F_{b,m,t}, Y_{r,j,t}, \\ Z_{j,m,t} \ge 0 \quad l = 1, \ldots, L \quad J = 1, \ldots, J \\ m = 1, \ldots, M \quad t = 1, \ldots, T \tag{2.12}$$

The objective function (2.1) minimizes the costs of liquefaction, re-gasification, and transportation. Equation (2.2) assures the liquid–gas balance. Equations (2.3, 2.4, 2.5, 2.6) assures the flow balance. Equation (2.7) guarantees that the liquid gas coming to the storage areas cannot exceed the capacity of the area. Equation (2.9) guarantees that the liquid gas supplied to the rented vessels cannot exceed the capacity of rented vessels. Equations (2.8, 2.10, 2.11) are demand constraints. Equations (2.12) are nonnegativity constraints.

3 Results for the model with deterministic demand

To understand the behavior of the system and test the solution reaction for different cases, 47 test problems are generated. For all of the parameters, uniformly distributed data are generated. In the following table, the objective function value and GAMS CPLEX solver's solution by using an Intel Core I5 4200U 2.30 GHz computer.

We see that increasing the number of customers and the facilities increase the costs in Table 2. For the 1st and 2nd cases, the objective function value of the second case is less than the first one although the number of storage areas is greater than the second case. Due to the high cost of the rented vessel, storage areas are primarily used. This is explained in the same way in 16–17 and 32–33. Therefore, when the capacity of the storage areas is fully utilized, after then the rented vessels are used. The objective function values of the cases 46–47, 42–43, 30–31, 26–27 were equal. The reason is that these pairs differ only from one parameter. However, this difference does not lead to full consumption of the capacity. Therefore, the model has chosen the lowest cost in any case and reached the same objective function values. In general, as a result of parameter changes, evaluations for t = 6,9,12 are made, and the rankings of each set in the relation are the same. Considering the solution time, it was found that there is no significant difference in all cases.

In Table 3, ten scenarios are evaluated. 10 new cases have been created. In these new cases, although the number of customers is few, the amount of demand is quite high. For example, as the time dimension increases, the value of the goal function has increased.

In Fig. 2, as the number of t periods increased, the value of the objective function value increases since the demand increases. However, in the cases with the same parameter values, different objective function values are obtained. In Table 4. decision variables values such as $O_{l,k,t}$, $BA_{l,b,t}$ and $Q_{l,r,t}$ are shown where t = 3.

When the 1st and 2nd cases are compared, although the total demand is the same, the objective function values are different since the cost of off-shore/ rented vessels (or floating holding tanks) is more than the cost of on-shore stationary LNG storage facilities.

For the second case, the reason for this is the reduction of the capacity of the on-shore stationary LNG storage facility b has been reduced. As a result of this, k rented vessels have been used. The use of k rented vessels increases the cost. The same is true for cases 4 and 5. The difference between the 3rd and the 4th case is that the customer g has been increased from 5 to 10.

In Fig. 3 total cost and the amount of LNG transported to off-shore/ rented vessels (or floating holding tanks), on-shore stationary LNG storage facilities, and to the regasification plants are shown.

Figure 3 shows that the total cost (objective function value) line is parallel to the amount of liquid gas transported from the liquefaction plant l to the off-shore/ rented vessel (or floating holding tank) k in the period t ($Q_{l,r,t}$) line. It is observed that the $Q_{l,r,t}$ decision variable is dominant on the objective function because the liquefied gas is regasified at the r regasification plant and distributed to the customers as NG. The volume of NG that is converted from liquid to gas increases the total cost (the objective function value). Therefore, this model also confirms that the use of LNG decreases costs.

Figure 4 shows the relation between the total LNG demand and the LNG storage in off-shore/ rented vessels

Table 2 Test problem results for the proposed model

Case	l	k	b	r	j	g	m	t	The objective function value ($x10^9$)	CPLEX solver solution time (sec)
1	4	4	3	5	3	10	10	6	29.05	0.02
2	4	4	4	5	3	10	10	6	28.59	0.02
3	4	4	5	5	3	10	50	6	29.50	0.02
4	4	4	5	5	3	10	70	6	29.88	0.01
5	4	4	10	5	3	10	50	6	29.43	0.05
6	4	4	10	5	3	10	100	6	30.60	0.05
7	4	4	10	5	3	50	50	6	160.21	0.02
8	4	4	10	5	3	50	100	6	161.37	0.05
9	10	4	10	5	3	50	100	6	161.18	0.11
10	10	4	10	5	3	50	200	6	163.51	0.11
11	10	4	10	5	3	100	200	6	314.70	0.12
12	20	4	10	5	3	100	200	6	314.51	0.11
13	20	4	15	5	3	100	300	6	316.40	0.27
14	20	4	15	5	3	200	300	6	625.45	0.34
15	30	5	20	5	3	200	500	6	629.22	0.99
16	4	4	3	5	3	10	10	9	49.11	0.09
17	4	4	4	5	3	10	10	9	49.11	0.13
18	4	4	5	5	3	10	50	9	49.27	0.11
19	4	4	5	5	3	10	70	9	49.37	0.09
20	4	4	10	5	3	10	50	9	49.27	0.09
21	4	4	10	5	3	10	100	6	49.48	0.16
22	4	4	10	5	3	50	50	9	254.63	0.09
23	4	4	10	5	3	50	100	9	254.97	0.13
24	10	4	10	5	3	50	100	9	254.85	0.11
25	10	4	10	5	3	50	200	9	255.27	0.14
26	10	4	10	5	3	100	200	9	509.06	0.14
27	20	4	10	5	3	100	200	9	509.06	0.14
28	20	4	15	5	3	100	300	9	509.48	0.17
29	20	4	15	5	3	200	300	9	1017.68	0.19
30	30	5	20	5	3	200	500	9	1036.41	0.31
31	30	10	20	5	3	200	500	9	1036.41	0.34
32	4	4	3	5	3	10	10	12	70.24	0.02
33	4	4	4	5	3	10	10	12	70.24	0.02
34	4	4	5	5	3	10	50	12	70.59	0.04
35	4	4	5	5	3	10	70	12	70.70	0.05
36	4	4	10	5	3	10	50	12	70.46	0.05
37	4	4	10	5	3	10	100	12	70.74	0.06
38	4	4	10	5	3	50	50	12	363.91	0.05
39	4	4	10	5	3	50	100	12	364.19	0.06
40	10	4	10	5	3	50	100	12	364.19	0.08
41	10	4	10	5	3	50	200	12	364.97	0.11
42	10	4	10	5	3	100	200	12	725.85	0.10
43	20	4	10	5	3	100	200	12	725.85	0.09
44	20	4	15	5	3	100	300	12	726.50	0.25
45	20	4	15	5	3	200	300	12	1449.54	0.25
46	30	5	20	5	3	200	500	12	1476.50	0.67
47	30	10	20	5	3	200	500	12	1476.50	0.59

Table 3 Test problems for 10 new cases

Case	l	k	b	r	j	g	m	t	The objective function value ($x10^9$)
1	3	3	3	3	2	5	5	3	181.23
2	3	3	3	3	2	5	5	3	181.51
3	3	3	3	3	2	5	10	3	182.53
4	3	3	3	3	2	10	10	3	379.80
5	3	3	3	3	2	10	10	3	386.18
6	4	3	3	3	3	10	20	6	577.77
7	4	3	3	3	3	20	20	6	1122.48
8	4	3	3	3	3	20	20	9	1612.72
9	4	3	3	3	3	20	20	9	1650.80
10	4	3	3	3	3	20	20	12	3185,30

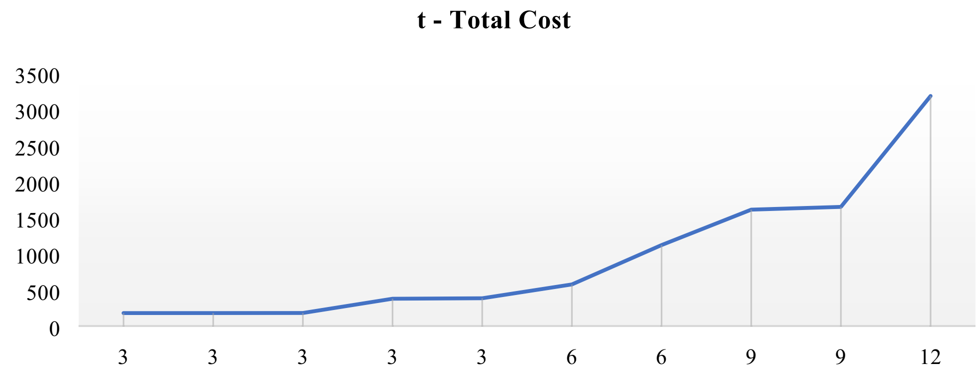


Fig. 2 t—Objective function values (total cost values) versus t periods

Table 4 Comparison of variables for t = 3

Case	$O_{l,k,t}$	$BA_{l,b,t}$	$Q_{l,r,t}$	Total	Actual demand for liquefied natural gas	Actual demand for natural gas
1	0	74,453.00	21,013.60	95,466.60	74,453.00	21,013.60
2	3741.00	70,712.00	21,013.60	95,466.60	74,453.00	21,013.60
3	0	156,133.00	21,013.60	177,146.60	156,133.00	21,013.60
4	0	156,133.00	41,982.21	198,115.21	156,133.00	41,982.21
5	114,298.00	41,835.00	41,982.21	198,115.21	156,133.00	41,982.21
6	0	617,276.00	80,910.15	698,186.15	617,276.00	80,910.15
7	0	617,276.00	158,753.44	776,029.44	617,276.00	158,753.44
8	0	915,457.00	242,618.21	1,158,075.21	915,457.00	242,618.21
9	774,257.00	141,200.00	242,618.21	1,158,075.21	915,457.00	242,618.21
10	0	1,790,586.00	498,324.36	2,288,910.36	1,790,586.00	498,324.36

(or floating holding tanks) and off-shore/ rented vessels (or floating holding tanks). In the 2nd, 5th, 9th cases there exist a need for off-shore/ rented vessels (or floating holding tanks) which will increase the cost while increasing the capacity and being used to satisfy the customer demand.

4 The model with uncertain demand

In this model, we assume that the demand of LNG for customer m in period t, (DD_{mt}), and demand of NG for customer g in period t, (DD_{gt}), are independently and identically distributed. We assume that DD_{mt} and DD_{gt} are

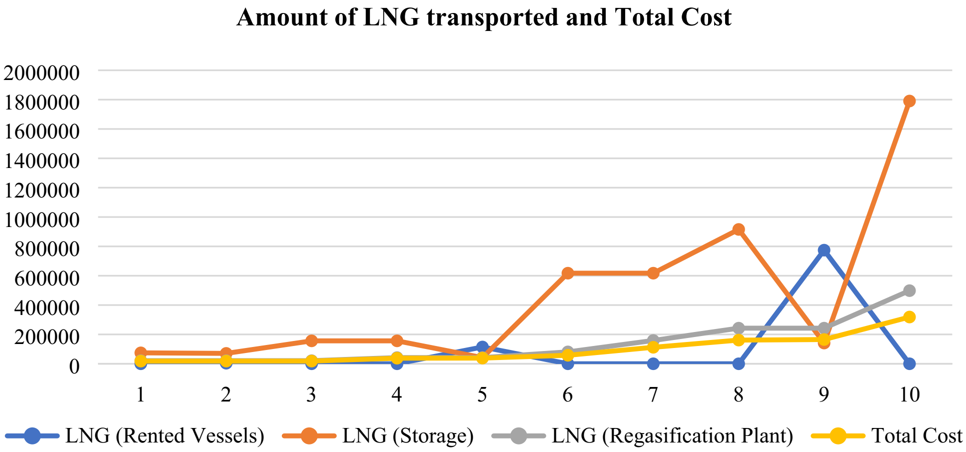


Fig. 3 Total cost and the relation between the amount of gas transported to off-shore/ rented vessels (or floating holding tanks), on-shore stationary LNG storage facilities, and to the regasification plants for all cases

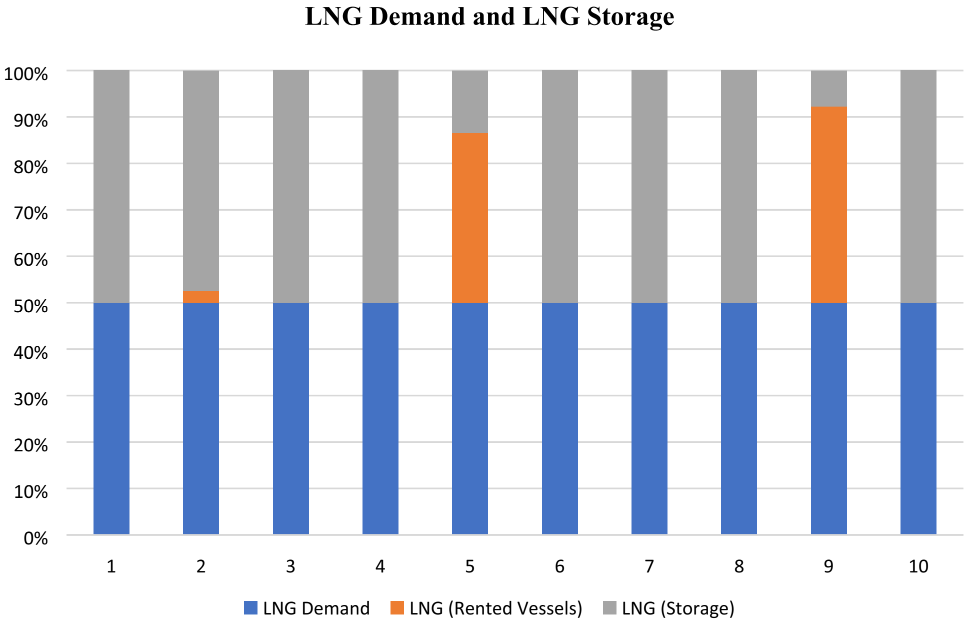


Fig. 4 The LNG demand and the relation between the amount sent to on-shore stationary LNG storage facilities and off-shore/ rented vessels (or floating holding tanks)

normally distributed with mean μ and standard deviation σ (Eqs. 4.1 and 4.2).

$$DD_{m,t} \sim N\left(\mu_{m,t}, \sigma_{m,t}\right) \tag{4.1}$$

$$DD_{g,t} \sim N\left(\mu_{g,t}, \sigma_{g,t}\right) \tag{4.2}$$

The Cycle Service Level (CSL) is defined as the probability of not having any shortage in a replenishment cycle [23]. In this model, we consider the LNG and NG demands which satisfy a CSL of 0.90. Denoting the safety factors by $\tau_{m,t}$ and $\tau_{g,t}$ we assume that the demand data are normally distributed, for 0.90 CSL, $\tau_{m,t} = 1.28$ and $\tau_{g,t} = 1.28$. Hence, the sum of the expected demands during θ periods and the required safety stock will be as in the formulas 4.3 and 4.4.

$$DS_{m,t} = \theta\mu_{m,t} + \tau_{m,t}\sigma_{m,t}\sqrt{\theta} \quad \mathrm{m} = 1, \ldots, \mathrm{M} \quad \mathrm{t} = 1, \ldots, \mathrm{T} \tag{4.3}$$

for the normally distributed demand for LNG with mean $\mu_{m,t}$ and standard deviation $\sigma_{m,t}$.

and

$$DS_{g,t} = \theta\mu_{g,t} + \tau_{g,t}\sigma_{g,t}\sqrt{\theta} \quad \mathrm{g} = 1, \ldots, \mathrm{G} \quad \mathrm{t} = 1, \ldots, \mathrm{T} \tag{4.4}$$

for the normally distributed demand for NG with mean $\mu_{g,t}$ and standard deviation $\sigma_{g,t}$ over θ consecutive periods.

In this model, the constraints (2.8), (2.10) and (2.11) in the deterministic model are modified with the constraints

$$\sum_{k} X_{\mathrm{k,m,t}} + \sum_{b} F_{\mathrm{b,m,t}} \geq DS_{m,t} \quad \mathrm{m} = 1, \ldots, \mathrm{M} \quad \mathrm{t} = 1, \ldots, \mathrm{T} \tag{4.5}$$

Table 5 Test problems for 10 cases for uncertain model

Case	l	k	b	r	j	g	m	t	The objective function value ($x10^9$)
1	3	3	3	3	2	5	5	3	194,19
2	3	3	3	3	2	5	5	3	195,48
3	3	3	3	3	2	5	10	3	212,80
4	3	3	3	3	2	10	10	3	403,08
5	3	3	3	3	2	10	10	3	410,83
6	4	3	3	3	3	10	20	6	709,68
7	4	3	3	3	3	20	20	6	1251,85
8	4	3	3	3	3	20	20	9	1949,79
9	4	3	3	3	3	20	20	9	2449,75
10	4	3	3	3	3	20	20	12	3590,47

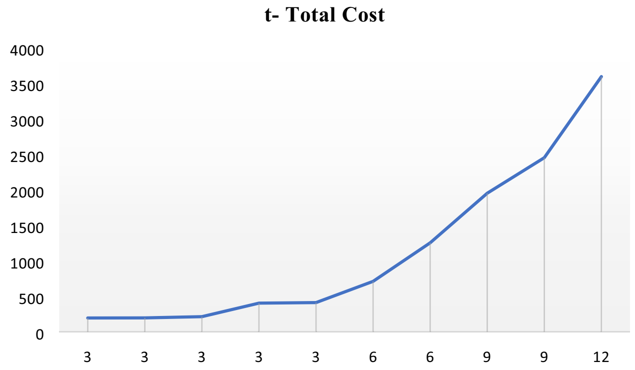


Fig. 5 t—Objective function values (total cost values) versus t periods

$$\sum\nolimits_j Z_{j,g,t} \geq DS_{g,t} \quad g = 1,\ldots,G \quad t = 1,\ldots,T \tag{4.6}$$

$$\sum\nolimits_l P_{l,,t} \geq \sum\nolimits_m DS_{m,t} + \sum\nolimits_g DS_{g,t}/600 \quad t = 1,\ldots,T \tag{4.7}$$

5 Results for the model with uncertain demand

In this part, $\mu_{m,t}$, $\mu_{g,t}$, $\sigma_{m,t}$ and $\sigma_{g,t}$ are generated with a normal random variable with hypothetic mean and variance values to obtain demand data which satisfies a CSL of 0.90. To understand the behavior of the system and test the solution reaction for different cases, 10 test problems are generated in the same way. In Table 5, the objective function value and GAMS CPLEX 24.1.3 solver's solution by using an Intel Core I5 4200U 2.30 GHz computer are stated.

According to the results stated in Table 5 the change in the objective value through period t is shown in Fig. 5.

In Fig. 5, as the number of periods increases the demand increases, and thus, the value of the objective function value increases. However, in the cases with the same parameter values, different objective function values are obtained. The reason for this is the use of rented vessels due to insufficient storage capacity. This is also true

Table 6 Comparison of variables for t = 3

Case	$O_{l,k,t}$	$BA_{l,b,t}$	$Q_{l,r,t}$	Total	Actual demand for liquefied natural gas	Actual demand for natural gas
1	0	90,064.41	15,695.34	105,759.75	90,064.41	15,695.34
2	24,135.61	65,928.80	15,695.35	105,759.76	90,064.41	15,695.35
3	0	184,839.45	16,877.27	201,716.72	184,839.45	16,877.27
4	0	184,839.45	33,076.78	217,916.23	184,839.45	33,076.78
5	143,004.45	41,835.00	33,076.78	217,916.23	184,839.45	33,076.78
6	0	1,519,693.82	136,747.53	1,656,441.35	1,519,693.82	136,747.53
7	0	1,519,693.82	274,932.37	1,794,626.19	1,519,693.82	274,932.37
8	0	2,582,985.97	414,630.45	2,997,616.42	2,582,985.97	414,630.45
9	1,170,975.97	1,412,010.00	414,630.45	2,997,616.42	2,582,985.97	414,630.45
10	0	3,340,670.75	552,399.97	3,893,070.72	3,340,670.75	552,399.97

for the deterministic model. In Table 6. decision variables values such as $O_{l,k,t}$, $BA_{l,b,t}$ and $Q_{l,r,t}$ are shown where t = 3.

Let's consider the 1st row in Table 6 for t = 3. There is no transfer to the rented vessels, but it is transferred to the storage area and regasification facility. The parameters used are: $\mu_{m,t=2000}$ and $\sigma_{m,t=250.}$ The results in the table indicate that the assignment should be according to the values in Table 5 considering the total demands for CSL = 0,90.

Figure 6 shows that the total cost (objective function value) line is parallel to the amount of liquid gas transported from the liquefaction plant l to the off-shore/ rented vessel (or floating holding tank) k in the period t ($Q_{l,r,t}$) line. It is observed that the $Q_{l,r,t}$ decision variable is dominant on the objective function because the liquefied gas is regasified at the r regasification plant and distributed to the customers as in gaseous form (NG). At the same time, as we can see it clearly in case 9, the contribution of $O_{l,k,t}$ variable to the cost function is quite high.

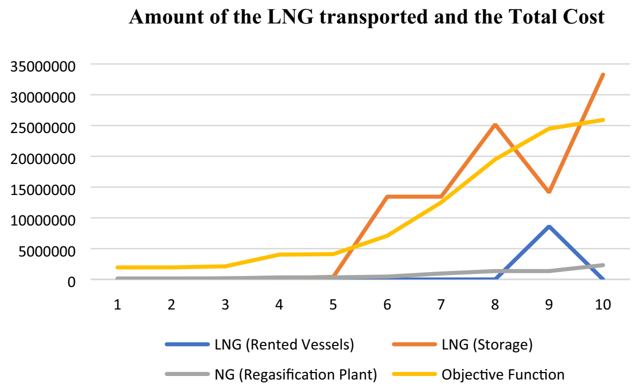


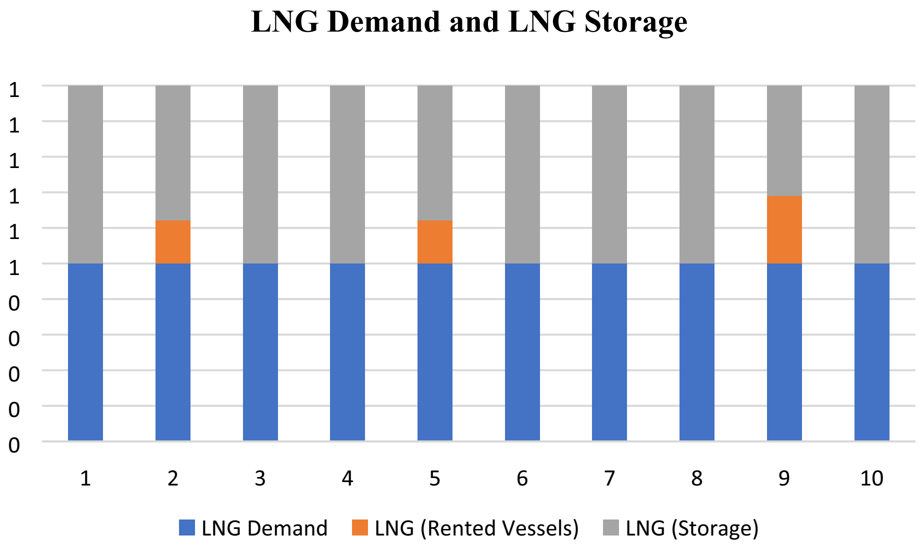
Fig. 6 Total cost and the relation between the amount of gas transported to off-shore/rented vessels (or floating holding tanks), on-shore stationary LNG storage facilities, and the regasification plants for all cases

Figure 7 shows the relation between the total LNG demand and the LNG storage in off-shore/ rented vessels (or floating holding tanks) and off-shore/ rented vessels (or floating holding tanks). In the 2nd, 5th, 9th cases, there exist a need for off-shore/ rented vessels (or floating holding tanks) which will increase the cost while increasing the capacity and being used to satisfy the customer demand.

6 Discussion

The developed model is different from the models in the literature regarding the transportation modes, storage types, capabilities, phases, and the demand patterns included in the model. We have included sea, road, and pipeline transportation modes. In the model, we have added on-shore stationary LNG storage facilities and off-shore/ rented vessels (or floating holding tanks). The off-shore/ rented vessels are used as floating storage areas including holding tanks in it when the on-shore stationary LNG storage facility capacity was exceeded. In none of the studies in the literature, off-shore/ rented vessels are used. When the studies are evaluated in terms of phases of the supply chain, some studies deal with just one phase of the supply chain while some studies deal with some of the phases. Most studies focus on three phases of the supply chain. Some of the studies are interested in liquefaction plant-storage- regasification plant, some of them are interested in liquefaction plant-storage-customer others are interested in liquefaction plant- regasification plant-customer. Different from the studies in the literature, our study focuses on liquefaction plant-storage-regasification

Fig. 7 The LNG demand and the relation between the amount sent to on-shore stationary LNG storage facilities and off-shore/ rented vessels (or floating holding tanks)



plant-customers including hubs. Thus, in this respect, this study is more complex than other studies in the literature.

In the second model, the demand uncertainty is included by adding normally distributed demand data. To cope with the uncertainty that arises with the probabilistic demand, we consider a CSL of 0.90 and include the necessary demand value for the test problems generated. Table 6 includes the amounts to be assigned to the rented vessels, storage areas, or the regasification facility for CSL = 0.90. The same situation in the deterministic model is given in Table 4. When the differences between assignments are evaluated, the following results are obtained. As the number of periods increases, the difference increases. The reason for this excessive increase is the addition of uncertainty and safety inventory to the deterministic model.

In this study, two different models are developed and tested for the same situations. Although the graphics given for the same conditions do not show similar behavior, they have some relations. In Fig. 8, we see that the biggest difference between the results of the two models is the cost. The CSL of 0.90 considers the safety stock which increases the cost to decrease the risk for the shortage. Although the demands are close to each other in both models, the second model adds the safety stock to the demand in order to satisfy the demand with 0.90 of CSL. The model which considers the demand uncertainty is more suitable for real-life problems because the demand behavior in real conditions demands are uncertain.

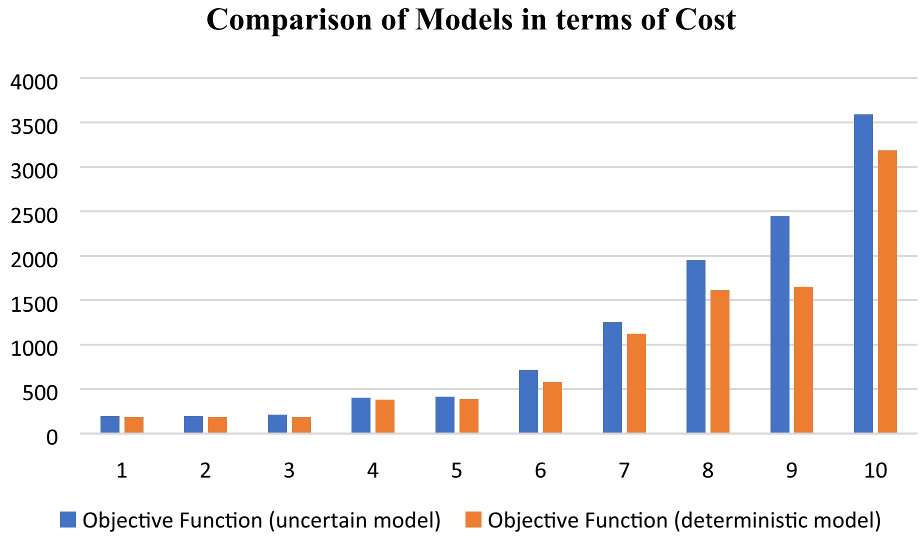
By using different test problems, we have observed that the decision variable which resembles the amount of liquid gas transported from the liquefaction plant to the off-shore/ rented vessels (or floating holding tanks) ($Q_{l,r,t}$) is the most efficient on the objective function because the volume of NG that is converted from liquid to gas increases the total cost (the objective function value). Therefore for the LNG supply chain, can say that the use of LNG decreases costs. Supplying the end-user by cryogenic trailers will be profitable.

A comparison of the studies in the literature and the proposed model in this study is summarized as follows:

When studies are examined in terms of modes, the studies [1, 2, 5, 7–16, 21] include only one mode. Studies [3, 4, 6, 17–20] include two modes of the transportation types. The studies examined were also evaluated in terms of storage areas. Studies 16 and 21 do not have a storage area. Most studies include one storage area. For example Studies [1, 2, 4–7, 9–15, 17–19] have one storage area. Studies [3, 8, 10] have two storage areas.

Unlike other models, our study includes off-shore/ rented vessels (or floating holding tanks) in addition to the on-shore/stationary storage facilities at the storage sites. The off-shore/ rented vessels and their usage together with on-shore/stationary storage facilities aren't used in any of the studies in the literature. When the studies are evaluated on a staged basis, the results are as follows. Sangaiah et al. [21] include one phase and Bittante [16] includes two phases. Studies [2, 8–12, 15, 17–20] include three phases. Studies [1, 3–7, 14] include four phases. Our study has liquefaction plant-storage-regasification plant-customers with three of the transportation modes included. The proposed model contains hubs that are important for the distribution of NG to the end-users via pipelines.

Fig. 8 Comparison of models in terms of cost



7 Conclusion

With the increasing population, energy consumption is increasing. Therefore, companies are focusing on the costs that will incur in the energy supply chain. The LNG is a major catalyst for any of the countries for industrial development and economic growth. For most of the countries, NG is the main feedstock for the country's energy needs. The use of NG will not only bring NG as a source of affordable energy to the countries but also create a culture of environmentally sustainable energy consumption among people which will reduce the deleterious effects of Heavy Fuel Oil (HFO), diesel and coal for generations to come.

In this study, the objective is to minimize the LNG supply chain costs by deciding where and how much NG/LNG inventory to keep on hand and where/how much to transfer the LNG and NG by way of different transportation modes. We propose two models with deterministic and uncertain demand behavior. Both models include liquefaction plants, storage areas, rented vessels, re-gasification plants, distribution hubs, and demand points. In the model, road, sea, and pipeline transportation modes are utilized. There are two types of customers in the study. The first one is the customer with LNG demand and the second one is the customer with NG demand. This is since, in some demand points, gas can be transported through the pipeline while in some demand points the gas cannot be transported due to the absence of a pipeline, the construction costs, and lack of capabilities.

To test the models, 67 test problems are randomly generated beginning from the simple to the complex. In the second model normally generated demand data are used. To cope with the uncertainty arising due to the probabilistic nature of the data a CSL of 0.90 is considered and the test problems are solved. The problems are solved by GAMS CPLEX 24.1.3 solver and we have obtained suitable solutions for the supply chain generated. Depending on the gas demand, the decisionmaker may have the opportunity to decide on whether there exists a need for an extra storage area by using rented vessels via this model while coping with the demand uncertainty. Especially, in high demand seasons, the LNG companies usually encounter such risky situations like gas shortage due to limited transportation and storage capacities. As a new alternative for fixed storage areas and tanks, the model decides to keep some of the LNG demand on the hired vessels. Rented vessels carrying portable storage tanks or rented LNG vessels will provide the companies to satisfy the customers and fix the costs of loss of goodwill. This alternative allows keeping the customer goodwill for future sales by satisfying their demands. On the other hand, in low demand seasons, there will be no need for extra capacities.

For the demand uncertainty, the second model adds the safety stock in order to satisfy the demand with 0.90 of CSL. While the decisionmakers guarantee not to have any stock out with 0.90 probability, they suffer much cost with the second model. Due to the demand behavior in real conditions demands are uncertain, the model which considers the demand uncertainty is more suitable for real-life problems.

For the LNG supply chain, the use of LNG decreases costs since the amount of transported gas increases, and the costs of regasification and transporting NG via pipelines decrease. The model confirms that the supply of the end-users by using cryogenic trailers will also be profitable.

The model includes the main hubs and the related pipeline network added to these hubs. By adding the facility location and the routing of the different types of vehicles for a defined road network and pipeline network, there may be a need to develop a heuristic solution method for the solution of large-scale LNG supply chain problems as a future study for this problem.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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